Influence of the Mechanical Parameters on the Forming Limit Curve

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Abstract

The aim of this paper is to evaluate the influence of the mechanical parameters on the uniaxial, plane strain and biaxial regions of the forming limit curve (FLC). The material used in this study is DC04 steel sheet. The experimental data needed for analyzing the variability of the mechanical parameters has been obtained from uniaxial tensile tests. The tests have been performed on samples cut at 0^o, 45^o and 90^o with respect of the rolling direction (RD). In this way, the yield stress and the plastic anisotropy coefficient have been determined for each of the orientations. The strain hardening exponent as well as the hardening coefficient for samples cut along RD has been also determined. Assuming that the mechanical parameters have a Gaussian distribution, an extension of their range to 3Sigma has been applied. In order to predict the FLC, a new formulation of the Modified Maximum Force Criterion is used. The influence of the mechanical parameters on the FLC is studied using the Taguchi method. By taking into account the variability of these parameters, an L12 orthogonal array is constructed. By applying the ANOVA procedure, the influence of each factor on the limit strains in the regions mentioned above is evaluated. A discussion referring to the amount of contributions is also made. Finally, a strategy for the determination of the forming limit band is proposed.

Keywords: Limit strains, Modelling, Taguchi method, ANOVA

Introduction

The FLC is a graphical representation of the strain states that produce the occurrence of a defect on the surface of the sheet metals (usually, necking or fracture). The experimental and theoretical research related to FLC's has been encouraged by their quick penetration in the industrial applications. The first experimental data referring to the FLC's were published by Keeler and Backofen (1964) and Goodwin (1968), respectively. The FLC concept is efficient in practice due to the fact that the limit strains can be easily measured. Its use in industry allows the prevention of material waste and the reduction of the times and prices related to the development of prototypes.

During the second half of the 20^{th} century, several models for the calculation of the limit strains have been developed. The theories proposed by Hill (1952) and Swift (1952) are based on the localized and diffuse necking hypotheses, respectively. Marciniak and Kuczynsky (1967) have proposed a model of strain localization based on the assumption that a thickness inhomogeneity exists from the very beginning of the forming process. In its original formulation, this model can be used only for calculating the tension-tension quadrant of the FLC. In order to extend the applicability of the Marciniak-Kuczynsky model to the tension-compression quadrant of the FLC, Hutchinson and Neale (1978) have developed a more general formulation that allows the planar rotation of the thickness defect. In 1975, Storen and Rice (1975) proposed the so-called "vertex theory" to model the localized necking under biaxial stretching conditions. Later on, Dudzinsky and Molinari (1991) proposed "the small perturbation theory" as a model of the plastic instability. Hora and Tong (1994) also developed the so-called Modified Maximum Force Criterion-MMFC with the aim of improving the diffuse necking model previously proposed by Swift. Their approach is based on the experimentally confirmed fact that the onset of necking significantly depends on the strain ratio. Recently, Hora's model has been improved by Hora and Tong (2006) and Comsa et al. (2010). An exhaustive description of the experimental and theoretical research on FLC's can be found in Banabic (2010a). A review of the Marcinik-Kuckzynski model is presented in Banabic et al. (2010b) and Banabic (2010c).

The mechanical parameters of the sheet metals have a strong influence on the forming limit curves. The first who noticed the variability of the experimentally determined FLC's were van Minh, Sowerby and Duncan (1974). After analyzing a large set of experimental results, they concluded that the scattering of the measured forming limits was caused by the errors in the experimental method and also by the variability of the material properties. The concept of Forming Limit Band (FLB) has been introduced by Janssens et al. (2001) on the basis of an experimental study referring to the accuracy of the FLC determination. By taking into account the variability of the mechanical parameters, lower and upper forming limit curves of the sheet metals can be drawn. Different approaches have been developed in order to predict the FLB. Banabic and Vos (2007) have used the Marciniak–Kuczynski model to calculate such FLB's. They determined the lower and upper forming limit curves by taking into account the variation of the parameters having control on the yield locus and the hardening rule of the metallic sheet.

The main purpose of this work is to analyze the influence of the mechanical parameters on the uniaxial, plane strain and biaxial regions of the forming limit curve. The influence of the parameters will be determined by applying the ANOVA method. Taguchi´s (1990) fractional factorial design of experiments is applied to plan the numerical simulations. The simulation plan is defined using an L12 Taguchi orthogonal array. The influence of the following mechanical parameters will be studied: the yield stresses and the anisotropy coefficients determined at 0^o, 45^o and 90^o with respect to the rolling direction. The strain hardening exponent of the power hardening law is also included in the analysis. At the end of the paper, a strategy for the determination of the forming limit band is proposed.

Prediction of the Forming Limit Curves

Throughout this paper, the sheet metal is considered to behave as an orthotropic membrane under the plane-stress conditions

$$\begin{aligned} &\sigma_{i3} = \sigma_{3i} = 0, \quad i = 1,2,3, \\ &\dot{\varepsilon}_{j3} = \dot{\varepsilon}_{3j} = 0, \quad j = 1,2, \end{aligned} \tag{1}$$

The above relationships involve the stresses and strain-rates expressed in the plastic orthotropy frame (1, 2 and 3 are the indices associated to the rolling, transverse, and normal directions, respectively). One also assumes that the external loads do not produce tangential stresses and strains:

$$\sigma_{12} = \sigma_{21} = 0, \quad \dot{\varepsilon}_{12} = \dot{\varepsilon}_{21} = 0 \tag{2}$$

The non-zero stresses and strain-rates thus become principal values. In order to emphasize their significance, the following notations will be used: $\dot{\varepsilon}_i = \dot{\varepsilon}_{ii}\left(i = 1,2,3\right)$ – principal strain rates, and $\sigma_j = \sigma_{jj}\left(j = 1,2\right)$ – principal stresses.

The mechanical response of the sheet metal will be described by a rigid-plastic model. The main ingredient of the constitutive model is the yield criterion:

$$\bar{\sigma}\left(\sigma_1, \sigma_2\right) = Y\left(\bar{\varepsilon}\right) \tag{3}$$

where $\bar{\sigma} = \bar{\sigma}\left(\sigma_1, \sigma_2\right) \geq 0$ is the equivalent stress (homogeneous function of the first degree), $\bar{\varepsilon} \geq 0$ is the equivalent strain, and ${}^tY = {}^tY\left({}^t\bar{\varepsilon}\right) > 0$ is the yield parameter controlled by a strictly increasing hardening law. The principal strain-rates are defined by the flow rule

$$\dot{\varepsilon}_j = \dot{\bar{\varepsilon}} \frac{\partial \bar{\sigma}}{\partial \sigma_j}, \quad j = 1,2, \tag{4}$$

and the incompressibility constraint

$$\dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 = 0 \tag{5}$$

In order to preserve the simplicity of the model, one assumes that the local state of the sheet metal evolves along linear load paths subjected to the constraint

$$\alpha = \sigma_2 / \sigma_1 = \text{const.}, \qquad \sigma_1 > 0, \quad \sigma_1 \geq \sigma_2 \tag{6}$$

For any load state having the property given by Equation (6), the equivalent stress and its partial derivatives with respect to the non-zero principal stresses could be expressed as follows:

$$\bar{\sigma} = \sigma_1 f\left(\alpha\right), \quad \frac{\partial \bar{\sigma}}{\partial \sigma_1} = g\left(\alpha\right), \quad \frac{\partial \bar{\sigma}}{\partial \sigma_2} = h\left(\alpha\right) \tag{7}$$

The functions f , g, and h are only related to the particular formulation of the equivalent stress adopted in the model. In this paper, the mechanical behavior of the sheet metal is described using the BBC2005 yield criterion (see Banabic 2010) yield criterion.

Equation (7) allows rewriting the yield criterion and the flow rule as follows (see Equations (3) and (4)):

$$\sigma_1 = Y\left(\bar{\varepsilon}\right) / f\left(\alpha\right), \tag{8}$$

$$\dot{\varepsilon}_1 = \dot{\bar{\varepsilon}} g\left(\alpha\right), \quad \dot{\varepsilon}_2 = \dot{\bar{\varepsilon}} h\left(\alpha\right) \tag{9}$$

One may prove that, under the constraint given by Equation (6), the strain path is also linear. As a consequence, Equation (9) can be easily integrated with respect to the time variable:

$$\varepsilon_1 = \bar{\varepsilon} g\left(\alpha\right), \quad \varepsilon_2 = \bar{\varepsilon} h\left(\alpha\right) \tag{10}$$

The FLC model used in this paper is a modification of Hora's model (1994) (**Fig. 1**).

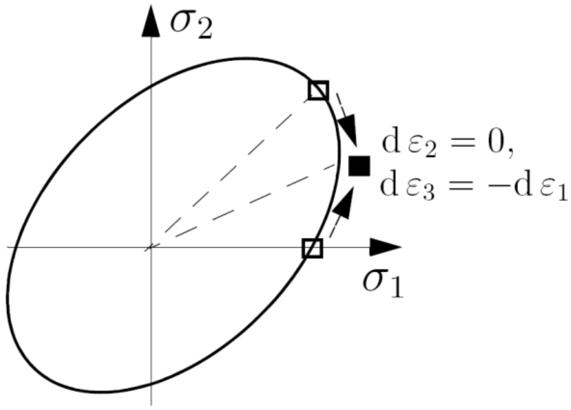

Figure 1. Evolution of the material towards the plane strain before the necking stage

According to Comsa et al. (2010), the necking occurs when the following equality is fulfilled:

$$\frac{\partial \sigma_1}{\partial \bar{\varepsilon}} \frac{\partial \bar{\varepsilon}}{\partial \varepsilon_1} + \frac{\partial \sigma_1}{\partial \gamma\left(\varepsilon_1, \alpha\right)} \frac{\partial \gamma\left(\varepsilon_1, \alpha\right)}{\partial \varepsilon_1} = \sigma_1, \tag{11}$$

In the relationship written above, $\gamma\left(\varepsilon_1, \alpha\right)$ is a measure of the "distance" separating the current state of the material from the plane-strain. The scalar quantity $\gamma\left(\varepsilon_1, \alpha\right)$ is defined by integrating the elementary arc-length of the normalized yield locus:

$$\mathrm{d}\,s = \sqrt{\left(\mathrm{d}\frac{\sigma_1}{Y}\right)^2 + \left(\mathrm{d}\frac{\sigma_2}{Y}\right)^2} \tag{12}$$

On the basis of the experimental evidence showing that the strain localization is preceded by the evolution of the material towards the plane-strain, the "distance" parameter is defined in the following manner:

$$\gamma\left(\varepsilon_1, \alpha\right) = \frac{1}{\varepsilon_1} \int_{\alpha_{FLC_0}}^{\alpha} \frac{\sqrt{g^2 + h^2}}{f^2} \mathrm{d}\underline{\alpha} \tag{13}$$

After some mathematical manipulations, Equation (11) can be rewritten in the form

$$Y=\frac{1}{g}\left(\frac{\mathrm{d}Y}{\mathrm{d}\bar{\varepsilon}}+\frac{Y}{\bar{\varepsilon}}\frac{f\,h}{\sqrt{g^2+h^2}}\int_{\alpha_{FLC_0}}^{\alpha}\frac{\sqrt{g^2+h^2}}{f^2}\mathrm{d}\underline{\alpha}\right) \quad (14)$$

This relationship allows the calculation of the equivalent strain associated to necking:

$$\bar{\varepsilon}=\frac{1}{g_\alpha}\left(n+\frac{f_\alpha h_\alpha}{\sqrt{g_\alpha^2+h_\alpha^2}}\int_{\alpha_{FLC_0}}^{\alpha}\frac{\sqrt{g_{\underline{\alpha}}^2+h_{\underline{\alpha}}^2}}{f_{\underline{\alpha}}^2}\mathrm{d}\underline{\alpha}\right) \quad (15)$$

As soon as $\bar{\varepsilon}$ is known, the corresponding principal strains result from Equation (10).

In order to build a reliable and robust FLB model, the influence of the constitutive equations on the limit strains should be analyzed. One possible approach consists in running the simulations by varying each mechanical parameter used in the identification procedures of the equivalent stress and hardening rule. If the equivalent stress has a complicated formulation, such a strategy becomes inadequate because the influence of each parameter on the yield locus cannot be intuitively deduced. With the aim of overcoming this difficulty, the authors have developed an alternative approach. One may notice that the lower level of the forming limit curve generally corresponds to the minimum values of the equivalent plastic strains defined by Equation (15). These minima are achieved when the following expression reaches its minimum value:

$$n+\frac{f_\alpha h_\alpha}{\sqrt{g_\alpha^2+h_\alpha^2}}\int_{\alpha_{FLC_0}}^{\alpha}\frac{\sqrt{g_{\underline{\alpha}}^2+h_{\underline{\alpha}}^2}}{f_{\underline{\alpha}}^2}\mathrm{d}\underline{\alpha} \quad (16)$$

i.e. when the hardening exponent n and the second term have the lowest values. The upper level of the forming limit curve corresponds to the opposite case, when both the first and second term in Equation (16) has maximum values.

Experimental Framework

The sheet metal used in the experiments is DC04 carbon steel with 0.85mm thickness.

Janssens et al. (2001) specify that a confidence level of 99.5% for Gauss normal distribution can be attained by performing at least 30 tests for each direction in the plane of the sheet metal. By using this assumption, in order to determine the variability of the mechanical parameters, 42 tensile tests have been performed using samples cut at 0^{o} with respect to the rolling direction. Additionally, 33 tensile tests have been made using samples cut at 45^{o} with respect to the rolling direction and 38 tensile tests using samples cut along the transverse direction.

The mechanical parameters used in this study are the yield stresses and the anisotropy coefficients, as well as the n coefficient of the power hardening law (Hollomon). The following statistical parameters have been calculated for these quantities: mean value of the noise variables that describes the central location of the data, and the standard deviation that shows the variation or "dispersion" of the experimental data from the mean value.

Table 1 shows the characteristic values of the mechanical parameters obtained from experiments, together with their minimum and maximum values. The statistical coefficients mentioned above have been calculated for each parameter.

Table 1. Mechanical parameters of the DC04 steel sheet (0.85 mm thickness).

Material parameter	Minimum value	Maximum value	Mean value	Standard deviation
Y_0[MPa]	190.56	198.98	195.96	2.086
r_0	1.72	2.20	1.92	0.110
Y_{45}[MPa]	207.06	215.35	210.97	2.401
r_{45}	1.17	1.44	1.31	0.062
Y_{90}[MPa]	201.75	209.79	205.49	2.154
r_{90}	2.00	2.65	2.22	0.145
n_0	0.20	0.21	0.21	0.002
K_0[MPa]	519.40	531.88	526.97	3.800

Taguchi method

The variability of the mechanical parameters is assumed to obey the Gauss normal distribution. Using this assumption, two levels of the parameters can be established (see **Table 2**). The first level is calculated by subtracting 3Sigma from the mean value, while the second level results by adding 3Sigma to the mean value.

Table 2. Mechanical parameters and their reference levels.

Mechanical parameters	Level 1	Level 2
n	0.2037	0.216
Y_0[MPa]	189.7	202.22
r_0	1.59	2.25
Y_{45}[MPa]	203.77	218.18
r_{45}	1.13	1.51
Y_{90}[MPa]	199.03	211.96
r_{90}	1.78	2.66

Table 3. . L12 orthogonal array and the model response for three regions of limit strains (BT – biaxial traction, PS – plane strain, UT – uniaxial traction).

Nr.	n	Y_0	R_0	Y_{45}	R_{45}	Y_{90}	R_{90}	BT	PS	UT
1	1	1	1	1	1	1	1	0.352	0.195	0.837
2	1	1	1	1	1	2	2	0.342	0.195	0.888
3	1	1	2	2	2	1	1	0.346	0.195	0.999
4	1	2	1	2	2	1	2	0.352	0.195	0.899
5	1	2	2	1	2	2	1	0.346	0.195	0.999
6	1	2	2	2	1	2	2	0.347	0.195	1.050
7	2	1	2	2	1	1	2	0.359	0.208	1.062
8	2	1	2	1	2	2	2	0.347	0.208	1.059
9	2	1	1	2	2	2	1	0.354	0.208	0.868
10	2	2	2	1	1	1	1	0.372	0.207	0.982
11	2	2	1	2	1	2	1	0.364	0.207	0.849
12	2	2	1	1	2	1	2	0.364	0.207	0.912

In order to optimize the number of numerical simulations, Taguchi's method (1990) has been adopted. This method uses an orthogonal array with combinations of possible conditions. In this study, an L12 orthogonal array has been chosen in order to establish the input of the numerical simulations. The objective of the computations evaluate the influence of the mechanical parameters on the uniaxial, plane strain and biaxial regions of the forming limit curve predicted by the MMFC criterion. The calculated values of the limit strains determined for each set of input data are listed in **Table 3**.

Influence of the mechanical parameters on the computed FLC's

The influence of each mechanical parameter on the FLC has been studied by using the analysis of the variance (ANOVA). Three regions of the FLC have been taken into account: biaxial traction (BT), plane strain (PS) and uniaxial traction (UT). The importance degree of each parameter is presented in **Table 4**.

Table 4. Percent contributions of each mechanical parameters on the limit strains corresponding to the biaxial, plane strain and uniaxial regions of the FLC

Material parameter	BT%	PS%	UT%
n_0	50.85	99.8975	0.2954
Y_0	18.91	0.03082	-0.0184
r_0	1.02	0.0347	86.273
Y_{45}	-0.12	-0.00010	0.201
r_{45}	6.413	0.003	0.432
Y_{90}	18.058	0.028	-0.025
r_{90}	3.4856	0.0027	12.000
Error -other parameters	1.369	0.0019	0.841

As one may notice, the hardening exponent has the strongest influence (about 50%.) in the biaxial region. The yield stresses corresponding to the rolling and transverse directions have a small influence (about 18%).

As expected, at the level of the plane strain region, the hardening exponent also has the strongest influence (more than 99%).

In the case of the tension-compression quadrant, the coefficients of plastic anisotropy r_0 and r_{90} have the most important effect on the limit strains: (about 86% and 12%, respectively).

Fig. 2 shows the influence of mechanical parameters on the yield locus predicted by the BBC2005 yield criterion. The scattering that can be noticed in the biaxial region (see the detail) is a consequence of the fact that the identification procedure used only uniaxial material data.

Fig. 3 shows the forming limit curves obtained by varying the mechanical parameters according to the Taguchi technique. In the plane strain region, the curves are grouped by the reference levels of the hardening exponent n. The scattering is more visible in the uniaxial and biaxial regions (see the detailed views at the top of **Fig. 3**.)

Conclusion

This paper illustrates the use of the MMFC model in connection with the Taguchi and ANOVA methods for analyzing the influence of seven material parameters on the forming limit curves. The mean values and the reference levels of the mechanical parameters have been determined from the results of uniaxial tensile tests performed at 0°, 45° and 90° with respect to the rolling direction.

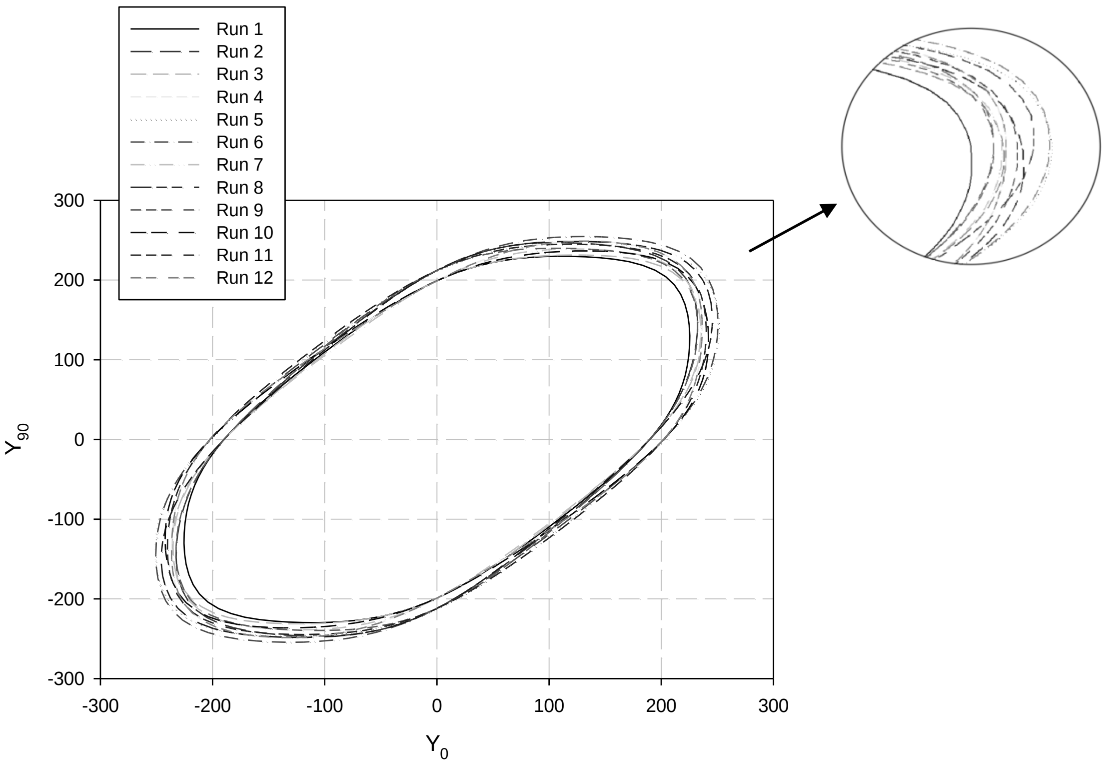

Figure 2 Yield loci obtained by varying the mechanical parameters according to the Taguchi technique

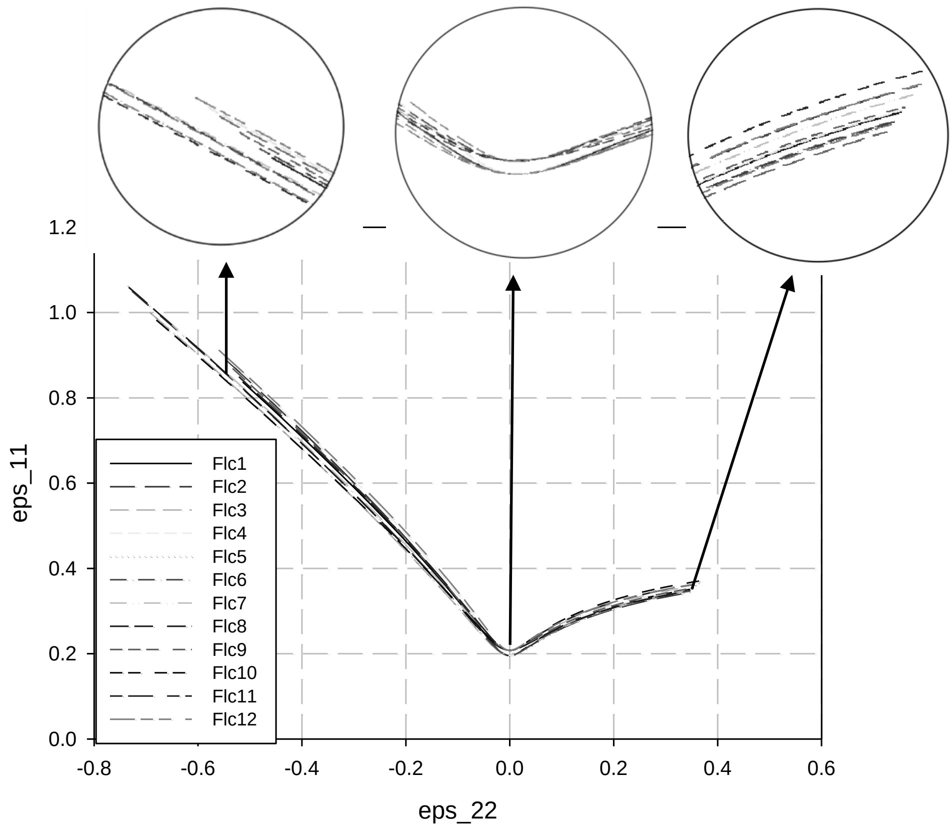

Figure 3 Forming limit curves obtained by varying the mechanical parameters according to the Taguchi technique

The influence of the mechanical parameters in three regions of the forming limit curve has been studied. The analysis shows that the hardening exponent has the strongest influence both in the biaxial and the plane strain regions. As concerns the left branch of the forming limit curve, the coefficients of plastic anisotropy r_0 and r_{90} are of greater importance.

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